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LOCAL GOVERNMENT REVENUE FORECASTING METHODS: COMPETITION AND COMPARISON

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ABSTRACT. This study examines forecast accuracy associated with the forecast of 55 revenue data series of 18 local governments. The last 18 months (6 quarters; or 2 years) of the data are held-out for accuracy evaluation. Results show that forecast software, damped trend methods, and simple exponential smoothing methods perform best with monthly and quarterly data; and use of monthly or quarterly data is marginally better than annualized data. For monthly data, there is no advantage to converting dollar values to real dollars before forecasting and reconverting using a forecasted index. With annual data, naïve methods can outperform exponential smoothing methods for some types of data; and real dollar conversion generally outperforms nominal dollars. The study suggests benchmark forecast errors and recommends a process for selecting a forecast method.

INTRODUCTION

To prepare budgets, governments forecast both revenue and expenditures. Within the United States, budgeting practices were introduced in the late 1800s and early 1900s to add a planning process before appropriating (Cleveland, 1913; Clow, 1896, 1901; Rubin, 1993; Williams, 2003). Part of the planning process is the identification of resource availability and resource constraint. This is the purpose for which governments make revenue forecasts in budgeting. Governments also forecast to track their progress compared with appropriations as the fiscal year progresses. Typically, these types of forecast are viewed as occurring in a unified process;

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however, this unification is practical, not essential. To track progress, it is necessary to have forecasts that reflect periodic expectation. Budgeting, on the other hand, requires only that there is an estimate for the entire fiscal period. Some evidence provided here reflects the use of annual forecasts, which may be useful for budgeting, but not for tracking. Governments may also forecast for long-term (beyond the next budget year) planning. No evidence in this study relates to such long-term forecasting.

In the United States there are 39,000 general purpose local governments and another 50,000 special purpose governments (National League of Cities, 2013). Only 98 of the general purpose governments have a population of 200,000 or more. Most have 25,000 or fewer. There are the 1261 localities with population between 25,000 and 200,000. Many of these medium sized localities have multiple – or even many – revenue sources. Yet, most of them – as well as some larger localities and likely all smaller localities – have limited resources to forecast their revenues. For these localities, it is beneficial to determine the effectiveness of forecasting methods that are relatively easy to perform. For this study, easy-to-perform is operationalized as (1) possible to implement in a spreadsheet following step-by-step guidance or (2) the product of forecast software that can be used out-of-the-box with limited knowledge about the techniques.

The primary aim of the study is to determine, from the examined methods, which are effective for forecasting these revenue data. Variability, seasonality, and sensitivity to economic factors can differ for different types of revenue. For example, among the series examined here, property tax data typically exhibits two very large seasonal peaks, which follows the typical local government practice of requiring semi-annual property tax payment, while sales tax data typically exhibits three small and one somewhat larger quarterly peaks. These types of revenue may have comparatively small random variation. Other types of revenue may exhibit proportionally larger random variation and little or no observable seasonality. Overall, forecasters may consider revenue data well behaved because they typically exhibit regular seasonal patterns if any, and limited growth. Consequently, forecasting should be relatively accurate. A secondary aim of this study is to address how accurate end users should expect revenue forecasts to be.

LITERATURE REVIEW

Forecast competitions (which contrast results from various forecasters) and comparisons (which contrast results from various methods) are commonly used to establish evidence based forecasting practices (Athanasopoulos, Hyndman, Song, & Wu, 2011; Ben Taieb, Bontempi, Atiya, & Sorjamaa, 2012; Gencay & Yang, 1996; Hong, Pinson, & Fan, 2014; Imhoff & Paré, 1982; Makridakis et al., 1982; Makridakis et al., 1993; Makridakis & Hibon, 2000). This literature sometimes examines domain specific forecasting methods (Chen, Bloomfield, & Cubbage, 2007; Chen, Bloomfield, & Fu, 2003). While methods researchers may use this approach to identify cutting edge methods (Makridakis et al., 1982; Makridakis et al., 1993; Makridakis & Hibon, 2000), domain-focused researchers frequently include a variety of relatively simple techniques and few complex methods (Chen et al., 2007; Chen et al., 2003; P. Sahu & Kumar, 2013; P. K. Sahu & Kumar, 2014). Within the domain of state and local government, similar research has focused on relative revenue forecast accuracy (Cirincione, Gurrieri, & van de Sande, 1999; Frank, 1990, 1993; Frank & Gianakis, 1990; Frank & Wang, 1994; Gianakis & Frank, 1993). These local government studies have used data from 1 to 8 localities within one state (Florida for the Frank studies, Connecticut for the Cirincione study), typically comparing under 10 techniques ranging in mathematical complexity from judgmental to ARIMA (Autoregressive Integrated Moving Average, also known as Box-Jenkins). This study extends the evidence based literature by examining data from 18 localities dispersed over 14 states,[1] and by comparing 37 different approaches to forecasting. It recommends a procedure for selecting specific forecasting methods considering the data's seasonality type, periodicity, and revenue type.

Revenue forecast literature frequently identifies the presence of underestimation bias (Bretschneider & Gorr, 1992; Bretschneider, Gorr, Grizzle, & Klay, 1989; Burkhead, 1956; Frank & Zhao, 2009; Klay & Grizzle, 1992; Rodgers & Joyce, 1996; Williams, 2012; Williams & Onochie, 2013), which is often treated as a rational hedge against uncertainty. Williams & Onochie (2013) and Levine, Rubin, and Wolohojian (1981) suggest that hedging may also be associated with increasing discretion during the post-appropriation period. For revenue forecasts, uncertainty is primarily associated with the lack of direct control and with forecast error. Although not commonly labeled as such, hedging through underestimation effectively results in overtaxing, underproduction of public services, or a combination of these. One way to reduce this negative consequence is to reduce uncertainty in another way, that is, by producing more accurate forecasts. However, there are technical limits to accuracy, which should be recognized in pursuit of improvement.[2] Another difficulty with reducing error is that more sophisticated techniques that typically produced lower error, including many examined here and particularly opaque software solutions, may be difficult to communicate in the context of a budget hearing.

Armstrong (2001) shows that forecast users value forecast accuracy above all other measured forecast characteristics. However, ease of implementation is also considered important; it is reasonable to anticipate that this characteristic is more important for moderately skilled forecasters. Complex methods are not included here. The study focuses on methods that are appropriate for moderately skilled forecasters. However, two types of automated forecast software, which may use complex methods, are included. Unique elements of this study include a focus on moderately skilled forecasters; use of local government revenue data; a focus on the budget period, which is typically the last 12 months of an approximately 18-month period; comparison of forecasts of annualized data with forecasts of monthly or quarterly data; comparison of forecasts in nominal or real dollars; and examination of a broad range of naïve techniques. All methods and approaches used in this study can be replicated by moderately capable users in a spreadsheet; they are not anticipated to be state-of-the-art methods.

STUDY DESIGN

This study compares a variety of forecast methods for 55 revenue series from 18 local governments dispersed over 14 states with a population ranging from 4 thousand to 1.4 million; half have populations between 25,000 and 200,000. There are 13 series each of property tax and sales tax, 10 total general fund series, and the remaining 19 series associated with a wide variety of revenue sources. The data include 4 annual series, 9 quarterly series, and 42 monthly series. The 4 annual series are all property tax. The series length (excluding holdout data of 18 months or two years) varies from 6 to 36 years, 16 to 154 quarters, and 54 to 183 months.

This analysis uses two approaches to making forecasts. First, the raw data, less holdout data, are submitted to Forecast Pro and Autobox software vendors[3] who are known to actively market forecast software to local governments (this inclusion is not endorsement or censure of these vendors). They are selected because they actively engage the forecast research community through their attendance at forecasting conferences and because they offer automated forecasting, which allows the moderately skilled forecaster to use their software.[4] These vendors are asked for an "automatic" result and a "best" result. The inclusion of the automatic result is to provide the potential user a suggestion of how well the software will perform with governmental revenue data for the moderately skilled user. The best results provide some insight into the additional gain that might be obtained by a highly skilled forecaster. The investigators also produced numerous forecasts of same series using widely recommended simple methods: moving averages, Simple Exponential Smoothing (SES, also called Single Exponential Smoothing or Exponentially Weighted Moving Average), Holt exponential smoothing (Holt, 1962), simplified Holt exponential smoothing, labeled TMW (T. M. Williams, 1987), and damped trend exponential smoothing (McKenzie & Gardner, 2010).[5] Exponential smoothing methods typically perform well in methods focused literature (Makridakis et al., 1982; Makridakis et al., 1993; Makridakis & Hibon, 2000). Five naïve methods that are anecdotally known to be used by moderately skilled forecasters are considered: (1) Last observation, also known as random walk or Naïve-1 (Hyndman, 2006), when deseasonalized, it may be labeled Naïve-2 (Makridakis, 1993); (2) Average of prior data; (3) Last change (rate of change in units) (Armstrong, 2001; Dalrymple, 1987); (4) Growth (rate of change as a ratio) (Armstrong, 2001; Dalrymple, 1987), may be labeled Naïve 2 in some forecast literature (Chen et al., 2007); and (5) time-index regression[6] (TIR) (Armstrong, 2001; Bretschneider & Gorr, 1999; Dalrymple, 1987; Guess, 2015; Mikesell, 2013; P. Sahu & Kumar, 2013). It is conventional wisdom among some forecast researchers that most of these sorts of naïve methods are likely ineffective (Armstrong, 2001, Principle 6.5), although TIR is sometimes described in methods textbooks. They are included here because they may naturally occur to moderately skilled forecasters. Some of these simple methods do not appear in prior studies. Causal methods are not included because of insufficient access to causal information. As part of this analysis, the quarterly and monthly series are annualized within the localities' fiscal years producing 55 total annual series.[7] Results across series are compared using the mean (across series) of the absolute percent error (MAPE), which is appropriate because the series differ in magnitude.

Three sorts of data preparation are used. First, extreme observations are modified to less extreme values using a process labeled Windsorizing (Armstrong, 1985). The data are filtered in a spreadsheet identifying any observations that are greater than 4 standard deviations from the mean of the series and moving the value to the 4 standard deviation upper boundary. No adjustment is made for lower boundary extreme values, which are typically bounded at zero (five series have 1 to 3 negative values). All techniques described below use the adjusted data. Second, except with annual data, the adjusted data are deseasonalized (Armstrong, 2001, Principle 5.5)[8] using classic decomposition, a technique commonly found in forecast literature (Miller & Williams, 2003; Williams, 2008). For all techniques except those labeled "Naïve Methods," all series are deseasonalized and some alternatives with each naïve method is deseasonalized. Actual deseasonalization can lead to any of three results: (1) multiplicative deseasonalization, (2) additive deseasonalization, or (3) determining that the data are not seasonal. The best method is selected by computing the absolute first differences for each form of deseasonalization (including no deseasonalization) and selecting the method with the smallest average. (Annual data are implicitly deseasonalized. For the other data, only one is determined to be nonseasonal.)

Third, an alternative considered uses real dollar data, adjusted by the CPI (Seasonally adjusted Consumer Price Index – All Urban Consumers CUSR0000SA0, 1950 to 2015) before forecasting and reversed after forecasting (Armstrong, 2001, Principle 5.1). To use only the CPI data that can be in hand at the time of the forecast, CPI is forecast at each possible revenue forecast origination date using Holt and SES. Although some moving averages are considered, they never generate the lowest RMSE. The real data thus computed and reconverted to nominal data are used exclusively by the investigators. (Local governments may further benefit by obtaining inflation forecasts from a reputable national firm. They may also prefer an alternate deflator; however, the selection should not be linked to government expenditures.) Real data are treated as separate, alternate data series;

thus, for every method used there is a forecast for the deseasonalized data and a forecast for the deseasonalized and real data.

For each naïve method, four forecasts are made: (1) Windsorized raw data, (2) deseasonalized data, (3) real data, and (4) deseasonalized real data. For moving average and exponential smoothing, two models are made for each method or combination of methods, (1) deseasonalized data and (2) deseasonalized real data.

In part of this study, the monthly and quarterly data are aggregated to the annual level and forecast for comparison between forecasting annual data and forecasting detailed periodic data before aggregating (Armstrong, 2001, Principle 2.2). An underlying objective of this study is to determine which approach or approaches provide the best budget forecast (an annual forecast). Periodic forecasts may have random variation that is reduced when summarized at the annual level; thus the average periodic error may be larger than the average error summarized at the annual level. However, forecasting periodic data and summarizing at the annual level provides more observations, which may lead to better model fitting than forecasting at the annual level. On the other hand, forecasting at the annual level eliminates concerns about seasonality or excessive deseasonalization in the data.

Forecasts are made using the actual length of the available data series excluding holdout data of 18 months, 6 quarters, or 2 years, which are used for effectiveness evaluation. (The Autobox forecaster reports that he truncated longer series.) To emulate the problem that local governments have in estimating their budget the data are compared with holdout data for months 7 through 18, quarters 2 through 6, or year 2 (Armstrong, 2001, Principle 13.5). The absolute value of the aggregate error for the entire emulated budget period is computed and compared using the mean of these absolute errors (MAPE), that is, the forecast is summed first, then the error is computed. The MAPE for this study is computed across forecasts, using aggregate errors computed across time.

RESULTS

Tables 1 - 5 compare MAPE for all forecasts produced for the study. Table 1 has two row sections, (1) the MAPEs and (2) the ranks of those MAPEs. Beginning with Table 2, a third section shows underperformance, which is the percent point difference between the

MAPE and the best MAPE for any method tested. Within each section the data are divided into four groups: (1) All series, (2) three seasonality types, (3) four tax types, and (4) three levels of periodicity (these reflect three different ways of subdividing the same data). For this study, MAPE is a measure of risk associated with using the forecast method and is likely more relevant to these data. Table 6 through 8 show summarized results and results related to annualized data. For all tables, the MAPE and APE is computed for months 7 through 18, quarters 2 through 6 or year 2. Frank and Zhao (2009) report that 80% or more of end users prefer an annual forecast error below 5%, mostly preferring error below 3%. As shown in these tables, the 3% threshold may be unrealistic. An average error below 5% is reasonably good. These error rates are computed on summed annual values of forecasts and actuals, when the absolute error for each period is computed then averaged, it is substantially larger.

Table 1 shows two groups: trending (1a) and growth (1b). These perform very poorly. For example, the top left cell of Table 1a (Section 1.1) shows that for all data types using the original data (no preprocessing except outlier adjustment) the average error for simple trending is greater than 1000%. In the same relative cell in Section 1.2, this approach (simple trending, limited preprocessing), is ranked 32nd out of 37 series.

TABLE 1
MAPE for Naïve Methods with Trend

Table 1.1: percent Table 1.2 rank		Trending Naïve Methods								
		1a. Trending				1b. Growth				
		Original Data†	Real	Deseas.		Original Data	Real	Deseas.		N‡
				Nominal	Real			Nominal	Real	
1.1 MAPE	All series	>1000	>1000	713.76	751.09	>1000	>1000	>1000	>1000	55
	Additive Seas.	>1000	>1000	701.18	741.22	>1000	>1000	>1000	>1000	20
	Multiplicative Seas.	>1000	>1000	722.15	757.67	>1000	>1000	>1000	>1000	30
	Nonseasonal	53.05	60.44			286.80	283.74			5
	Sales Tax	>1000	>1000	198.44	202.07	>1000	>1000	>1000	>1000	13
	Property Tax	>1000	>1000	>1000	>1000	>1000	>1000	>1000	>1000	13
	Other Revenue	634.93	659.00	519.25	545.44	>1000	>1000	>1000	>1000	19
	Total General Fund	>1000	>1000	517.56	525.01	>1000	>1000	>1000	>1000	10
	Annual & Annualized	9.59	9.86			10.49	10.62			55
	Quarterly	98.04	98.36	107.32	106.19	>1000	>1000	79.12	80.23	9
	Monthly	>1000	>1000	846.88	892.65	>1000	>1000	>1000	>1000	42

TABLE 1 (Continued)

Table 1.1: percent Table 1.2 rank		Trending Naïve Methods								
		1a. Trending				1b. Growth				
		Original Data†	Real	Deseas.		Original Data	Real	Deseas.		N‡
				Nominal	Real			Nominal	Real	
1.2 Rank MAPE	All series	32	33	30	31	34	35	37	36	
	Additive Seas.	32	33	30	31	36	37	34	35	
	Multiplicative Seas.	32	33	30	31	34	35	37	36	
	Nonseasonal	24	25			27	26			
	Sales Tax	32	33	30	31	35	34	36	37	
	Property Tax	32	33	30	31	34	35	37	36	
	Other Revenue	32	33	30	31	36	37	35	34	
	Total General Fund	32	33	30	31	37	36	34	35	
	Annual & Annualized	15	17			20	22			
	Quarterly	32	33	35	34	36	37	24	25	
	Monthly	32	33	30	31	35	34	37	36	

Notes: †"Original Data" are adjusted to remove outliers. ‡The number of observations in deseasonalized naïve methods may be reduced by up to 5 observations when there is interaction with nonseasonal data.

- "Trending" – The last observation plus the change from the second last observation to the last observation.
- "Growth": The last observation multiplied by the rate of change from the last prior observation.
- "Deseas.": The data are deseasonalized as appropriate (automated software may deseasonalized differently than otherwise).
- "Real": Data are deflated using Consumer Price Index (CPI) before forecasting and re-inflated using a simple forecast of CPI as the final step of forecasting.
- "Nominal": Not "Real."
- "N": The number of series. This information is omitted for sections 2 and 3 of tables, because it doesn't change.
- "SES": Simple Exponential Smoothing.
- "Holt": Holt two parameter exponential smoothing.
- "TMW": An alternate to Holt using a simpler trend parameter.
- "DT": damped trend with Holt parameters.
- "DT TMW": Damped trend using the TMW trend parameter.
- "Autobox" and "Forecast Pro": Software sources.
- "Automatic": Software run in automatic mode.

Following the top row of Section 1.2 across, to the right, the worst performance of all approaches for all data (ranked 37 of 37) is the use of growth with deseasonalized nominal data as shown in the next to last column. Except with annual (or annualized) data, Section 1.1 shows extremely high MAPEs that would not be satisfactory for forecasting and Section 1.2 shows that the best rank for these is 15 out of 37, indicating that 14 other methods outperform the best of these results. Preprocessing data through deseasonalization or conversion to real dollars does not substantially improve these approaches and sometimes makes them worse.

There are 37 types of forecast compared with all data except nonseasonal and annual data, where there are 27 and 24 types, respectively (the mid-ranks are 19, 14 and 12.5). Table 1.2 shows that these trending methods rank 30-37; for nonseasonal data, 24-27; and for annual/annualized data, 16-24; so, except with annual data, they always have the worst forecast errors. The errors are far larger than any reasonable tolerance level. With annual data, they are in the bottom half of performance, although not the worst. Based on this evidence, these methods should never be used for medium term forecasting of revenue data.

Table 2 shows the non-trend (Panels A and B) and the time index regression naïve methods (Panel C). Section 2.1 of each panel shows that these methods perform substantially better than the trend methods shown in Table 1, with MAPE values ranging from 4.21% (nonseasonal/original data/last observation) to 92.16% (original data/quarterly/average of all). However, Section 2.2 shows that, excluding nonseasonal data and annual/annualized data (which are naturally nonseasonal), these methods dominate the remainder of the low-performing methods; always ranking 16 or higher. For the average of all series (Section 2.2, top row) the lowest (best) rank is 16 for time index regression using deseasonalized nominal data and the highest is 29 for last observation using real data. With the highest rank of 16, only two of the methods not labeled naïve perform worse. The two types of data for which they rank higher are nonseasonal and annual. For the nonseasonal data, the MAPE can be quite good with a value of 4.21% (Panel A Section 2.1 nonseasonal/original data/last observation). However, this result ranks 7 (Panel A Section 2.2), so six methods outperform it. Last observation also performs well with annual/annualized data, ranking first with real data and second with the original data (Panel A Section 2.2). Excluding these two types of data, Panel A Section 2.3 shows that the methods underperform (are worse than the best method) by 3.78 to 92.16 percent points.

TABLE 2
MAPE: Naïve Methods

Tables 2.1 and 2.3: percent Table 2.2: rank		Original Data	Real	Deseas. Nominal	Deseas. Real	N
Panel A. Last Observation						
2.1 MAPE	All series	41.40	41.56	21.11	17.07	55
	Additive Seas.	60.16	60.43	17.92	16.54	20
	Multiplicative Seas.	35.09	34.96	23.24	17.43	30
	Nonseasonal	4.21	5.67			5
	Sales Tax	45.24	44.72	20.73	12.86	13
	Property Tax	58.02	59.54	29.13	28.90	13
	Other Revenue	24.92	24.71	20.21	16.75	19
	Total General Fund	46.12	46.09	16.00	12.49	10
	Annual & Annualized	6.98	6.82			55
	Quarterly	88.54	88.38	21.84	9.87	9
	Monthly	34.93	35.07	20.95	18.65	42
2.2 Rank MAPE	All series	28	29	21	19	
	Additive Seas.	28	29	21	19	
	Multiplicative Seas.	29	28	21	19	
	Nonseasonal	7	11			
	Sales Tax	29	27	22	16	
	Property Tax	27	29	25	24	
	Other Revenue	23	22	21	19	
	Total General Fund	29	28	23	17	
	Annual & Annualized	2	1			
	Quarterly	27	26	23	21	
	Monthly	26	27	25	23	
2.3 Dif. From Best	All series	35.28	35.44	14.99	10.96	
	Additive Seas.	52.75	53.02	10.50	9.13	
	Multiplicative Seas.	29.61	29.48	17.76	11.94	
	Nonseasonal	0.97	2.44			
	Sales Tax	41.26	40.74			
	Property Tax	52.63	54.15	23.74	23.51	
	Other Revenue	17.56	17.35	12.86	9.39	
	Total General Fund	41.50	41.47	11.39	7.87	
	Annual & Annualized	0.16	0.00			
	Quarterly	84.26	84.10	17.56	5.59	
	Monthly	28.40	28.53	14.42	12.12	
Panel B. Average of All Observations						
2.1 MAPE	All series	29.10	27.32	20.88	16.82	55
	Additive Seas.	30.20	29.17	17.59	16.19	20
	Multiplicative Seas.	29.81	27.80	23.08	17.25	30
	Nonseasonal	20.39	17.03			5
	Sales Tax	40.87	38.14	21.16	13.31	13
	Property Tax	26.44	24.69	28.54	28.28	13
	Other Revenue	28.87	28.29	19.85	16.35	19

TABLE 2 (Continued)

Tables 2.1 and 2.3: percent Table 2.2: rank		Original Data	Real	Deseas. Nominal	Deseas. Real	N
	Total General Fund	17.68	14.84	15.47	11.92	10
	Annual & Annualized	15.53	12.79			55
	Quarterly	92.16	90.86	21.84	9.87	9
	Monthly	16.02	14.71	20.67	18.35	42
2.2 Rank MAPE	All series	25	24	20	18	
	Additive Seas.	25	24	20	18	
	Multiplicative Seas.	27	23	20	18	
	Nonseasonal	23	22			
	Sales Tax	25	24	23	17	
	Property Tax	21	20	23	22	
	Other Revenue	27	26	20	18	
	Total General Fund	25	19	22	16	
	Annual & Annualized	24	23			
	Quarterly	31	30	22	20	
	Monthly	19	18	24	22	
2.3 Dif. From Best	All series	22.98	21.21	14.77	10.71	
	Additive Seas.	22.79	21.76	10.18	8.78	
	Multiplicative Seas.	24.33	22.32	17.59	11.76	
	Nonseasonal	17.15	13.80			
	Sales Tax	36.89	34.16	17.19	9.34	
	Property Tax	21.05	19.31	23.15	22.89	
	Other Revenue	21.51	20.93	12.49	8.99	
	Total General Fund	13.06	10.23	10.86	7.31	
	Annual & Annualized	8.71	5.97			
	Quarterly	87.88	86.58	17.56	5.59	
	Monthly	9.49	8.18	14.14	11.82	
Panel C. Time Index Regression Methods						
2.1 MAPE	All series	25.33	26.42	15.53	16.28	55
	Additive Seas.	26.80	27.63	14.96	15.62	20
	Multiplicative Seas.	27.56	28.60	15.91	16.72	30
	Nonseasonal	6.04	8.53			5
	Sales Tax	44.23	45.21	16.84	17.70	13
	Property Tax	13.69	14.91	19.65	19.26	13
	Other Revenue	25.64	26.88	13.05	14.34	19
	Total General Fund	15.30	16.09	14.57	15.24	10
	Annual & Annualized	9.63	9.58			55
	Quarterly	90.38	90.34	8.06	8.29	9
	Monthly	13.21	14.39	17.16	18.03	42
2.2 Rank MAPE	All series	22	23	16	17	
	Additive Seas.	22	23	16	17	
	Multiplicative Seas.	22	25	16	17	
	Nonseasonal	13	19			
	Sales Tax	26	28	18	19	

TABLE 2 (Continued)

Tables 2.1 and 2.3: percent Table 2.2: rank		Original Data	Real	Deseas. Nominal	Deseas. Real	N
	Property Tax	16	17	19	18	
	Other Revenue	24	25	16	17	
	Total General Fund	21	24	18	20	
	Annual & Annualized	16	14			
	Quarterly	29	28	16	17	
	Monthly	16	17	20	21	
2.3 Dif. From Best	All series	19.21	20.31	9.41	10.16	
	Additive Seas.	19.39	20.22	7.55	8.21	
	Multiplicative Seas.	22.08	23.11	10.42	11.24	
	Nonseasonal	2.80	5.30			
	Sales Tax	40.25	41.23	12.86	13.72	
	Property Tax	8.30	9.52	14.26	13.87	
	Other Revenue	18.28	19.52	5.69	6.98	
	Total General Fund	10.69	11.47	9.96	10.63	
	Annual & Annualized	2.82	2.76			
	Quarterly	86.10	86.06	3.78	4.01	
	Monthly	6.68	7.86	10.63	11.50	

Notes: See Table 1's notes.

TABLE 3
MAPE for Moving Average Methods

Table 3.1 and 3.3: percent Table 3.2: rank		Moving Average				
		3a. Moving Average		3b. Moving Average with Tend		
		Nominal	Real	Nominal	Real	N
3.1 MAPE	All series	8.05	7.79	36.77	36.99	55
	Additive Seas.	9.29	9.03	56.68	55.26	20
	Multiplicative Seas.	7.76	7.06	28.03	29.12	30
	Nonseasonal	4.88	7.27	9.61	11.19	5
	Sales Tax	6.50	5.65	18.94	19.10	13
	Property Tax	7.31	8.61	56.89	58.40	13
	Other Revenue	9.43	8.89	33.36	32.39	19
	Total General Fund	8.44	7.43	40.29	41.16	10
	Annual & Annualized	8.51	8.34	9.10	10.13	55
	Quarterly	5.83	4.64	9.27	8.60	9
	Monthly	8.81	8.36	45.86	46.15	42
3.2 Rank MAPE	All series	13	12	26	27	
	Additive Seas.	10	8	27	26	
	Multiplicative Seas.	13	11	24	26	
	Nonseasonal	8	**18**	20	21	
	Sales Tax	**15**	9	20	21	
	Property Tax	10	14	26	28	

TABLE 3 (Continued)

Table 3.1 and 3.3: percent Table 3.2: rank		Moving Average				
		3a. Moving Average		3b. Moving Average with Tend		
		Nominal	Real	Nominal	Real	N
	Other Revenue	8	6	29	28	
	Total General Fund	**15**	14	26	27	
	Annual & Annualized	10	9	12	19	
	Quarterly	9	**2**	19	18	
	Monthly	13	12	28	29	
3.3 Dif. From Best	All series	1.94	1.68	30.66	30.88	
	Additive Seas.	1.88	1.61	49.27	47.85	
	Multiplicative Seas.	2.28	1.57	22.55	23.63	
	Nonseasonal	1.65	4.04	6.37	7.95	
	Sales Tax	2.52	1.67	14.96	15.12	
	Property Tax	1.92	3.22	51.50	53.01	
	Other Revenue	2.07	1.53	26.00	25.03	
	Total General Fund	3.82	2.81	35.67	36.55	
	Annual & Annualized	1.69	1.53	2.28	3.31	
	Quarterly	1.55	0.36	4.99	4.32	
	Monthly	2.27	1.83	39.33	39.61	

Notes: See Table 1's notes.

Table 3 shows MAPEs of moving average methods. Table 3 is divided into two sets: moving average (1a) and moving average with trend (1b). For the forecast methods reported in Tables 3-4, all series are deseasonalized as appropriate, so two variants of each technique are examined, one uses deseasonalized data alone, and the other used deseasonalized real data. Moving average is a commonly recognized simple forecast method. For implementation, its advantage is that the math can be learned in a few minutes. A disadvantage is that in a spreadsheet, parameter (length of the moving average) fitting requires re-writing the formula for each possible value (number of periods over which the moving average is calculated). Table 3b shows that the moving average with trend performs as worse than man of the naïve methods demonstrated in Table 2. For example, for all series real data, Table 3b Section 3.1 shows a MAPE of 36.99, which, Section 3.2 shows, is ranked 27, worse than all but two of the methods examined in Table 2. In fact, Section 3.2 shows that the moving average with trend always ranks among the bottom half of methods except with annual/annualized nominal data, where it is ranked 12th. Table 3a shows that the non-trending moving average performs relatively well.

For these forecasts, MAPE range from 5.64 (quarterly data) to 9.43 (other revenue). Table 3b Section 3.2 shows that for six of these data types, a moving average method is ranked among the top quarter of methods, for example the moving average for other revenue/real data is ranked 6th. For quarterly data, the moving average with real dollars is ranked second. Underperformance (Table 3a Section 3.3) ranges from 0.36 (other revenue/real) to 4.04 percent points (nonseasonal/real) for moving averages and 2.28 (annual/annualized/nominal) to 53.01 (property tax/real) percent points for moving average with trend.

Table 4 shows the exponential smoothing methods. Exponential smoothing requires modest skill with math. It typically uses two to four formulas to implement and another one to two formulas to initialize. The most difficult of these are, collectively, roughly comparable to deseasonalization. Table 4 is divided into three sets: SES methods (4a), Holt and TMW methods (4b) and damped trend methods (4c).

Table 4a Section 4.2 shows that SES/nominal performs best for series with additive seasonality and with nonseasonal data; and SES/real performs best with quarterly data and second best with sales tax. Table 4a Section 4.3 shows that the underperformance for sales tax is 0.07 percent points. For "All Series" SES is not among the top five methods. Table 4a Section 4.3 shows that with all series underperformance is 0.86 (nominal) to 1.07 (real) percent points. In the deeper details, underperformance ranges up to 4.40 percent points (property tax/real).

Table 4b shows that Holt and TMW methods perform similarly with each other, with the two comparable methods often ranked beside each other. For example, Table 4b Section 4.2 shows that for all series Holt/nominal is ranked 11th and TMW/nominal is ranked 10th. Use of nominal data typically ranks better than the comparable real dollar data, for example TMW/nominal ranks 10th and TMW/real ranks 15th; however, this is rank order is not consistent across all series types. These methods are ranked in the top 5 with only two series types, TMW/nominal/nonseasonal and Holt/real/total general fund, and never better than 4th. They underperform by 0.31 (TMW/nominal/nonseasonal) to 4.40 (TMW/real/other revenue) percent points; or averaged across all series, it ranges from 1.44 (TMW/nominal) to 2.47 (TMW/real) percent points.

Table 4c Section 4.2 shows that damped trend methods outperform their parallel Holt or TMW methods by about 1 percent point (except in one instance where TMW/nominal outperforms the parallel DT TMW/nominal by 0.05 percent for other revenue). Damped

TABLE 4
MAPE for Exponential Smoothing Methods

Table 4.1 and 4.3: percent Table 4.2: rank		Exponential Smoothing										N
		4a. SES		4b. Holt & TMW				4c. Damped Trend				
		Nom.	Real	Holt Nom.	Holt Real	TMW Nom.	TMW Real	DT Nominal	DT Real	DT TMW Nominal	DT TMW Real	
4.1 MAPE	All series	6.98	7.19	7.61	8.43	7.55	8.58	6.83	7.17	6.78	6.90	55
	Additive Seas.	7.41	8.44	10.08	9.49	10.31	9.99	9.08	8.57	9.39	8.15	20
	Mult. Seas.	7.31	6.57	6.60	8.02	6.38	7.94	5.87	6.41	5.58	6.18	30
	Nonseasonal	3.24	5.91	3.73	6.66	3.55	6.83	3.61	6.19	3.53	6.28	5
	Sales Tax	5.28	4.05	5.96	5.95	5.47	6.18	4.83	4.26	4.56	4.17	13
	Property Tax	6.92	9.79	6.44	7.31	7.30	8.49	5.39	6.28	6.09	7.75	13
	Other Rev.	8.55	8.32	10.2	12.44	9.86	11.76	9.73	10.97	9.91	9.22	19
	Total GF	6.26	5.73	6.28	5.52	6.20	5.77	5.81	4.90	4.62	4.95	10
	Annualized	7.50	7.28	8.95	10.02	9.31	10.54	7.88	7.57	7.83	8.16	55
	Quarterly	6.05	4.28	6.10	6.48	5.64	6.42	4.85	5.28	4.80	5.29	9
	Monthly	7.52	7.79	8.30	8.96	8.32	9.16	7.56	7.58	7.49	7.22	42
4.2 Rank MAPE	All series	7	9	11	14	10	**15**	4	8	**2**	5	
	Additive Seas.	**1**	6	14	12	**15**	13	9	7	11	5	
	Mult. Seas.	12	9	10	**15**	6	14	3	8	**2**	4	
	Nonseasonal	**1**	12	6	**16**	4	**17**	5	14	3	**15**	
	Sales Tax	7	**2**	11	10	8	12	6	4	5	3	
	Property Tax	8	**15**	5	11	9	13	**1**	4	3	12	
	Other Rev.	5	3	12	**15**	10	14	9	13	11	7	
	Total GF	10	6	11	5	9	7	8	**2**	**1**	3	
	Annualized	4	3	11	**18**	13	**21**	7	5	6	8	
	Quarterly	10	**1**	11	13	8	12	4	6	3	7	
	Monthly	6	9	10	14	11	**15**	7	8	5	4	
4.3 Dif. From Best	All series	0.86	1.07	1.49	2.32	1.44	2.47	0.72	1.06	0.66	0.79	
	Additive Seas.	0.00	1.03	2.67	2.08	2.90	2.58	1.67	1.16	1.98	0.74	
	Mult. Seas.	1.82	1.09	1.12	2.54	0.90	2.45	0.39	0.93	0.10	0.69	
	Nonseasonal	0.00	2.68	0.50	3.43	0.31	3.59	0.37	2.95	0.29	3.04	
	Sales Tax	1.30	0.07	1.98	1.97	1.49	2.20	0.85	0.29	0.58	0.20	
	Property Tax	1.53	4.40	1.06	1.92	1.91	3.10	0.00	0.89	0.70	2.36	
	Other Rev.	1.19	0.96	2.86	5.08	2.50	4.40	2.37	3.61	2.55	1.86	
	Total GF	1.64	1.12	1.67	0.90	1.59	1.16	1.20	0.29	0.00	0.33	
	Annualized	0.68	0.46	2.13	3.20	2.49	3.72	1.06	0.75	1.01	1.34	
	Quarterly	1.77	0.00	1.82	2.20	1.36	2.14	0.57	1.00	0.52	1.01	
	Monthly	0.99	1.26	1.77	2.42	1.79	2.63	1.03	1.05	0.96	0.69	

trend methods are frequently among the top 5 methods, with DT/nominal ranked first for property tax, and DT-TMW/nominal ranked first for total general fund. For other series, underperformance ranges up to 3.61 (DT/real/other revenue) percent points and the underperformance for "All series" ranges from 0.72 to 1.06 percent points.

Table 5 shows MAPEs for the three forecasts from software. Automated methods can deseasonalize, when appropriate. These methods do not include real dollar conversion. Two columns are labeled "Automatic" and one is labeled "Best." Forecast Pro said that adjusting away from automatic would provide little gain. The Autobox Best is identical to its Autobox Automatic except with 5 series. Comparing the results on the top row of Section 5.1 (all series) Autobox Best underperforms Autobox Automatic by 0.15 percent points. Section 5.2 shows that the software typically performs very well. For "All series," Forecast Pro ranks first, and Autobox Automatic ranks 3. The difference between these two vendors is 0.68 percent points. Autobox Automatic is top rank for other revenue, while Forecast Pro is ranked second. Autobox Best is ranked second for property tax. This is the only data type for which Autobox plus professional judgment outperforms Autobox by itself. Because there are only 5 series where professional judgment differed from the automatic output, there are many ties between these two methods. Forecast Pro is top ranked for multiplicative seasonality, sales tax, and monthly series. Section 5.2 shows software sometimes ranks comparatively low for property tax, sales tax, total general fund, multiplicative seasonality and nonseasonal series.

Table 6 summarizes the results from the prior tables showing best, second best (among all methods), and worst (among methods shown in Tables 3a, 4 and 5) MAPE values. Columns 2-3 show the best Method and its MAPE; columns 4-5 show the same information for the alternate method. Column 6-8 show the difference between best and the alternative method, which is second best in Section 6.1 and worst in Section 6.2. Section 6.3 compares all series with periodic data and with annualized data. Columns 9-12 compare these errors, showing the percent point difference between the two, the standard error of this difference, and a paired t-test statistical significance of this difference. Section 6.1 shows that there actual differences between the best and

TABLE 5
MAPE for Forecast Software

Tables 5.1 and 5.3: percent Table 5.2: rank		Forecast Software			N
		5. All Software			
		Autobox Automatic	Autobox Best	Forecast Pro Automatic	
5.1 MAPE	All series	6.79	6.94	6.12	55
	Additive Seas.	7.75	8.09	7.71	20
	Multiplicative Seas.	6.37	6.39	5.48	30
	Nonseasonal	5.51	5.64	3.52	5
	Sales Tax	6.31	6.31	3.98	13
	Property Tax	6.46	5.43	6.88	13
	Other Revenue	7.36	8.49	7.63	19
	Total General Fund	6.78	6.78	5.03	10
	Annual & Annualized†	4.17	4.33	3.70	4
	Quarterly	6.90	6.90	5.25	9
	Monthly	7.02	7.20	6.53	42
5.2 Rank MAPE	All series	3	6	**1**	
	Additive Seas.	3	4	**2**	
	Multiplicative Seas.	5	7	**1**	
	Nonseasonal	9	10	**2**	
	Sales Tax	13.5	13.5	**1**	
	Property Tax	6	**2**	7	
	Other Revenue	**1**	4	**2**	
	Total General Fund	12.5	12.5	4	
	Annual & Annualized				
	Quarterly	**14.5**	**14.5**	5	
	Monthly	**2**	3	**1**	
5.3 Dif. From Best	All series	0.68	0.83	0.00	
	Additive Seas.	0.34	0.68	0.30	
	Multiplicative Seas.	0.88	0.91	0.00	
	Nonseasonal	2.28	2.40	0.29	
	Sales Tax	2.33	2.33	0.00	
	Property Tax	1.07	0.04	1.49	
	Other Revenue	0.00	1.13	0.27	
	Total General Fund	2.17	2.17	0.41	
	Annual & Annualized				
	Quarterly	2.62	2.62	0.97	
	Monthly	0.49	0.67	0.00	

Notes: †Because the number of observations is limited to the originally annual data for this table, comparisons with other methods (Sections 5.2 and 5.3) are not reported. In Tables 1-4, annualized data are included; however, the annualized data were not considered by the software vendors.

second best method are small, never more than 0.7 percent points (property tax), and not statistically distinguishable except with property tax. Section 6.2 shows that among the better methods, on average the best method outperforms the worst by 2.47 (all series) to 5.08 (other revenue) percent points, and for all series types except sales tax, the results are statistically distinguishable. Section 6.3 compares the best results for each of two approaches used for all the series and shows that forecasting the periodic data outperforms the annualized data; however, the 0.70 percent point difference is not statistically distinguishable.

Table 6 also shows that for most series types, the most effective forecast is performed with nominal dollar data; however, with annual and quarterly data, it is more effective to use real dollar data. For annual data, the best method is last observation/real and the second best method is last observation/nominal.[9] As Section 6.3 shows, this result does not outperform use of the periodic data; however, the difference in MAPE is surprisingly small. Section 6.1 shows that for all series types, the best MAPE is less than 7.5. This is a potential target for forecast accuracy. However, this target should be used with caution, the maximum APE for the best method is 23.01 for "All series," and ranges from 8.32 (quarterly) to 32.37 (annualized & annual). These higher values arise because some individual series have more variance and unpredictable components

TABLE 6
Summarized Results

	Series Type	Best	MAPE	Alternate	MAPE	Dif. MAPE	SE	Sig.	N
6.1 Best vs. Second Best	All series	Forecast Pro Automatic	6.12	DT TMW Nominal	6.78	0.66	0.90		55
	Additive Seas.	SES Nominal	7.41	Forecast Pro Automatic	7.71	0.30	1.23		20
	Multiplica-tive Seas.	Forecast Pro Automatic	5.48	DT TMW Nominal	5.58	0.10	0.85		30
	Nonsea-sonal	SES Nominal	3.24	Forecast Pro Automatic	3.52	0.29	1.14		5
	Sales Tax	Forecast Pro Automatic	3.98	SES Real	4.05	0.07	1.17		13
	Property Tax	DT Nominal	5.39	DT TMW Nominal	6.09	0.70	0.49	*	13
	Other Revenue	Autobox Automatic	7.36	Forecast Pro Automatic	7.63	0.27	1.21		19

TABLE 6 (Continued)

	Series Type	Best	MAPE	Alternate	MAPE	Dif. MAPE	SE	Sig.	N
	Total General Fund	DT TMW Nominal	4.62	DT Real	4.90	0.29	0.85		10
	Annual & Annualized	Naïve Last Obs. Real	6.82	Naïve Last Obs. Nominal	6.98	0.16	0.21		55
	Quarterly	SES Real	4.28	MA Real	4.64	0.36	0.44		9
	Monthly	Forecast Pro Automatic	6.53	Autobox Automatic	7.02	0.49	0.90		42
6.2 Best vs. Worst	All series	Forecast Pro Automatic	6.12	TMW Real	8.58	2.47	1.11	**	55
	Additive Seas.	SES Nominal	7.41	TMW Nominal	10.31	2.90	1.86	*	20
	Multiplicative Seas.	Forecast Pro Automatic	5.48	Holt Real	8.02	2.54	1.15	**	30
	Nonseasonal	SES Nominal	3.24	Autobox Automatic	7.27	4.04	2.73	*	5
	Sales Tax	Forecast Pro Automatic	3.98	MA Nominal	6.50	2.52	2.01		13
	Property Tax	DT Nominal	5.39	SES Real	9.79	4.40	1.79	**	13
	Other Revenue	Autobox Automatic	7.36	Holt Real	12.44	5.08	2.21	**	19
	Total General Fund	DT TMW Nominal	4.62	MA Nominal	8.44	3.82	1.59	**	10
	Annual & Annualized	Naïve Last Obs. Real	6.82	TMW Real	10.54	3.72	1.06	***	55
	Quarterly	SES Real	4.28	Autobox Automatic	6.90	2.62	1.38	**	9
	Monthly	Forecast Pro Automatic	6.53	TMW Real	9.16	2.63	1.42	**	42
6.3 Comparing the best method for all series from two approaches.									
	All series/ Annualized	Forecast Pro Automatic Periodic	6.12	Naïve Last Obs. Real Annualized	6.82	0.70	1.05		55

Notes: *** = 0.001 ** =0.05 * =0.1. This table compares only the methods in Tables 3a, 4, and 5. Excludes Autobox Best as not necessarily indicative of results achievable by moderately skilled forecasters.

Abbreviated Column Labels for Tables 6 through 9: "Best" – Method with lowest MAPE. MAPE – Average APE calculated for each series. "Alternate" – The second best or the worst method as defined in text and footnotes. Dif. MAPE – (difference) the alternate MAPE minus the best MAPE. "SE" = Standard Error of the difference in MAPE. "Sig." The statistical significance of a paired t-test for the dfference in MAPE. "N" – The number of series.

TABLE 7
Annualized Data Summarized

	Series Type	Best	MAPE	Alternate	MAPE	Dif. MAPE	SE	Sig.	N
7.1 Best vs. Second Best	All series	Last Obs. Real	6.82	Last Obs. Nominal	6.98	0.16	0.21		55
	Additive Seas.	DT Real	6.30	DT TMW Real	6.71	0.41	0.35		20
	Multiplicative Seas.	SES Real	6.71	Last Obs. Real	7.03	0.32	1.24		30
	Nonseasonal	Growth Real	2.44	Growth Nominal	2.45	0.01	0.38		5
	Sales Tax	SES Real	6.24	SES Nominal	6.98	0.74	0.93		13
	Property Tax	DT Nominal	5.90	DT TMW Nominal	5.94	0.04	0.33		13
	Other Revenue	Last Obs. Real	6.95	Last Obs. Nominal	7.14	0.18	0.35		19
	Total General Fund	Last Obs. Real	3.79	DT TMW Real	4.17	0.38	0.81		10
	Annual	Growth Real	1.81	Nominal Growth	1.86	0.05	0.48		4
	Quarterly	Last Obs. Real	5.42	Nominal Last Obs.	6.14	0.71	0.33	**	9
	Monthly	SES Real	7.27	Last Obs. Real	7.35	0.08	0.76		42
7.2 Best vs. Worst	All series	Last Obs. Real	6.82	Average All Nominal	15.53	8.71	1.89	***	55
	Additive Seas.	DT Real	6.30	Average All Real	12.78	6.48	2.14	**	20
	Multiplicative Seas.	SES Real	6.71	Average All Nominal	16.81	10.10	2.22	***	30
	Nonseasonal	Growth Real	2.44	Average All Nominal	20.28	17.84	10.25	*	5
	Sales Tax	SES Real	6.24	MA Trend Real	11.22	4.98	3.96		13
	Property Tax	DT Nominal	5.90	Average All Nominal	20.58	14.68	4.48	**	13
	Other Revenue	Last Obs. Real	6.95	Average All Nominal	18.42	11.47	3.02	***	19
	Total General Fund	Last Obs. Real	3.79	Average All Nominal	9.24	5.45	2.50	**	10
	Annual	Growth Real	1.81	Average All Nominal	24.50	22.69	11.66	*	4
	Quarterly	Last Obs. Real	5.42	Average All Nominal	18.61	13.18	6.50	**	9
	Monthly	SES Real	7.27	Average All Nominal	14.02	6.74	1.57	***	42

Notes: *** = 0.001 ** =0.05 * =0.1. This table excludes forecast software methods. Periodicity is reported from the original data.

Table 7 further examines the annualized data. (In this table, annual series are those that were originally obtained as annual. Forecast software is not included in this analysis.) Section 7.1 shows that last observation/real is the most effective method for all series, other revenue, total general fund, and quarterly series types. Growth/real is the most effective method for the annual and the nonseasonal series; however, for these two series types, the number of observations is very small and four of the five nonseasonal series are the original annual series. For annualized data, real dollar data outperforms nominal dollar data for all series types except property tax. As with the periodic data, the two types of exponential smoothing that are most effective are damped trend and Simple Exponential Smoothing. For annual data, damped trend outperforms damped trend/TMW except with total general fund. Section 7.2 shows that for all series types except sales tax, the poorest performing method is the average of all data; and with the further exception of additive seasonal series, performance is poorest with nominal dollar data.

Table 8 compares the best method with periodic data to the best method with annualized data. Negative numbers in the “Dif. MAPE” column are associated with better performance with the annualized data (for the series annualized/annual, standard error and significance is not computed because the number of observations differs). For six series types the periodic data outperforms the annualized data. The performance difference ranges from 0.51 to 2.26 percent points. For the four series types where the annualized series outperforms the periodic series, the range is 0.41 to 1.11 percent points. None of these results are statistically significant. Although not statistically significant, it is interesting that for other revenue and total general fund, no method outperforms last observation/real with annualized data. The superior result for Growth/real for nonseasonal series is less interesting because the number of observations is very small. The practical implication of these results is that use of annual data may only marginally reduce forecast accuracy and may increase it in some cases.[10]

DISCUSSION

This study compares a large number of easy to modestly difficult forecast methods that can be implemented either with spreadsheets or with automated forecast software. The results show that there is no

TABLE 8
Periodic vs. Annualized

Series Type	Periodic	MAPE	Annualized	MAPE	Dif. MAPE	SE	N
All series	Forecast Pro Automatic	6.12	Real Last Obs.	6.82	0.70	1.05	55
Additive Seas.	SES Nominal	7.41	DT Real	6.30	**-1.11**	1.47	20
Multiplicative Seas.	Forecast Pro Automatic	5.48	SES Real	6.71	1.23	1.35	30
Nonseasonal	SES Nominal	3.24	Growth Real	2.44	**-0.80**	1.35	5
Sales Tax	Forecast Pro Automatic	3.98	SES Real	6.24	2.26	2.25	13
Property Tax	DT Nominal	5.39	DT Nominal	5.90	0.51	1.31	13
Other Revenue	Autobox Automatic	7.36	Last Obs. Real	6.95	**-0.41**	1.25	19
Total General Fund	DT TMW Nominal	4.62	Last Obs. Real	3.79	**-0.83**	1.30	10
Annualized/ Annual	Real Last Obs.	6.82	Growth Real	1.81			55:4
Quarterly	SES Real	4.28	Last Obs. Real	5.42	1.14	2.11	9
Monthly	Forecast Pro Automatic	6.53	SES Real	7.27	0.74	1.07	42

one approach that produces the most accurate forecasts in all instances. For periodic data, the most promising methods are simple exponential smoothing, damped trend and statistical software. Generally, it is not beneficial to preprocess periodic data converting nominal dollars to real dollars, except with quarterly or annual data. Most results suggest that simplified trend methods (Trending, Growth and Moving Average with Trend) should be avoided, notwithstanding the seemingly good results of Growth with annualized data for two series types that have 5 or fewer series. The study shows that the use of annualized data converted to real dollars may result in about the same level of accuracy as periodic data. As a practical matter, use of annualized data may be simpler than use of a deseasonalization method. The most likely methods for annualized data are damped trend, SES or last observation. For annualized data, forecast software was not included in the study. These results are mostly consistent with the view that naïve methods should be avoided, although they are less harmful with annualized data. The results suggest that forecasts should be tested with the benchmark last observation using holdout data before selecting a more sophisticated method.

Armstrong (2001) recommends a comprehensive set of practices (also labeled principals) for forecasting addressing such major topics as defining the problem, obtaining independence from politics; using appropriate data; cleaning data; selecting simple, unbiased, quantitative, causal methods, implementing the forecast conservatively; avoiding forecasts of cycles; and frequently updating. His recommendations examine the entire forecast process from planning to forecast through forecast evaluation. Following is a recommended decision process with respect to selecting simple unbiased quantitative methods using the empirical information reported here. It focuses on selecting the most likely effective simple method for the type of revenue data to be forecast and is generally consistent with Armstrong, except where previously noted. Use the following steps:

1. Choose and test a variety of methods:
   a. Choose both of the best overall methods: forecast software for the original periodic data and last observation for annualized data. Also choose software for the annualized data (although this option was not included in the study, the general software results support always considering software).
   b. Using the left column of Table 8, choose the best combination of forecast method and preprocessing associated with each of the three examined characteristics (seasonality type, revenue type and periodicity) for the original periodic data.
   c. Using the right column of Table 8, choose the best method and preprocessing for each of these characteristics for annualized data.
   d. If forecast software it is not available, choose the relevant second best method from Table 6 as a substitute.
   e. If you currently use a different approach, also choose this. In total you may have as many as 10 methods.
   f. If real dollar preprocessing is not recommended and the government subscribes to economic forecast of inflation, consider forecasting monthly real data otherwise using the approach from step b.
2. Exclude the last 18 months, 6 quarters or 2 years from the data. If local conditions indicate that the budget revenue forecast requires more or fewer months or quarters, adjust the excluded data to match local conditions. For annual or annualized data, always exclude 2 years, unless local conditions require more years.

3. If you have unique knowledge not captured in historical data, such as a policy change (e.g., a change in the tax rate) record its impact. Only record the information you would have had at the beginning of the holdout data (Armstrong, 2001, Principle 7.5). Do not interpret the actual data and correct the information, only record the sort of data you would also know in its next occurrence.
4. Using the necessary formulas found in the appendix both for preprocessing the data and for forecast, and using any guidance provided with the forecast software, make a forecast through the holdout period.
5. Sum the data to one value for 12 months, 4 quarters or 1 year.
6. Adjust for the information recorded in step 3 (Armstrong, 2001, Principle 7.5).
7. Using the formulas found in the appendix calculate the APE for each forecast.
8. Select the method with the lowest APE.
9. For monthly and quarterly data, repeat this process every quarter. For data that is only available annually, repeat it every year (Armstrong, 2001, Principle 13.13, 14.11, 16.3).
10. If after several quarters, the selected method cycles between two methods, it may be advisable to average them.
11. If more than two methods are commonly selected, you may need to seek expert forecasting advice.
12. If the best method uses annualized data and it is important to also track performance during the year, the best periodic method should also be used routinely.

LIMITATIONS

Some of the more significant limitations are: For some series types, the number of observations is very small and results should be viewed with substantial caution. Due to the limited number of observations, no results are reported at the interaction level (for example, multiplicative seasonality-property tax-monthly). Series examined were voluntarily provided by local governments and are not a random sample, thus statistical results should be viewed with caution. Annualized data and real dollar data were not forecast by forecast software vendors, so some unexamined combinations may provide additional gain in forecast accuracy. The reported results are computed as absolute forecast error, this measures accuracy, but it does not measure bias.

CONCLUSION

This study examines forecasts of 55 monthly, quarterly, and annual local government revenue data series from 18 localities. The data series include 13 property tax, 13 sales tax, 10 total general fund, and 19 of other types of revenue. There are 42 monthly series, 9 quarterly series, and 4 annual series. They include 30 series with multiplicative seasonality, 20 with additive seasonality, and 5 nonseasonal (including the 4 annual series). Data are preprocessed removing certain extreme observations, and sometimes preprocessed to deseasonalize and/or to convert nominal dollars to real dollars using CPI. The deseasonalized data are then forecast using 5 types of exponential smoothing and 2 types of moving average, each with and without the real dollar conversion. Four versions of the data (seasonal-nominal, deseasonal-nominal, seasonal-real, and deseasonal-real) are forecast using five naïve methods (last observation, average of all data, last change, growth, and time index regression). The quarterly and monthly series are also annualized and forecast with these methods (annual data are implicitly nonseasonal). The original data are provided to two vendors of automated forecasting software, both of whom provided output from their automated forecasts, and one of whom also provided a "best" forecast. Data are evaluated for the end year of an 18 month (or two year, for annual data) holdout period using APE, thereby simulating the actual revenue forecast problem for local governments.

The most effective methods for monthly or quarterly data are automated forecast software, damped trend, and SES. The strong performance of automated software and damped trend are not surprising; however, it is useful for forecasters to know that these methods outperform a much wider variety of simple methods than previously tested. The relatively strong performance of SES suggests that the revenue data may frequently reflect limited change over time. For monthly data, conversion to real dollars results in poorer forecast performance. This result could be an artifact of the methods used as both the deflator and the content data are estimated using similar forecasting methods. If the forecaster subscribes to a deflator forecast made using other methods, real dollar conversion may be more effective. For annual and annualized data, it is hard to outperform last observation; although for some segments of the data, damped trend and SES are more effective. However, with annual data it is effective to convert data to real dollars before forecasting. Forecast software was not tested for annualized data. Overall, use of periodic data slightly outperforms annualized data. A step-by-step process is recommended for the local revenue forecaster to choose a method for each data series.

By using actual local revenue series and focusing on a realistic revenue forecast horizon (months 7 through 18, aggregated), this study provides useful advice to revenue forecasters that is not available with studies that focus on other types of data, periodic error, or results that include horizons that are not within the likely budget period. The advice addresses many more methods than those reported in other revenue forecast studies. The advice is summarized in twelve recommended steps to selecting and reselecting the best method to use for specific series.

Many elements of this study are consistent with Armstrong's (2001) recommended forecasting principles. However, the results are ambiguous with respect to principle 5.1, which recommends removing inflation from data; obtained results are consistent with preferring nominal dollars with monthly data. The results are somewhat contrary to principle 6.5 when forecasting annual level data as very simple naïve methods frequently perform best. This study also contributes to cumulative knowledge about forecasting by establishing potential benchmarks for forecast accuracy for local government revenue data and by showing the relative effectiveness of various methods when used with these sorts of data.

The unexpected results related to the effectiveness of last observation with annualized data and ineffectiveness of real dollar conversion for monthly and quarterly data suggest a need for additional research with other data series. Some of the forecast competitions cited at the beginning of this article are completed without access to the actual dates of the data, so conversion to real dollars would not be possible. More studies should be conducted using not only actual data, but also data that has not been stripped of its actual dates.

NOTES

1. Specific localities are not identified as permission to identify was not provided at the time they voluntarily provided data to GFOA.

2. Techniques discussed here may also be used for some forms of expenditure forecasting; however, this sort of data is not evaluated. Some types of projections, particularly payroll expenditures and property tax, can be estimated through more deterministic methods when they involve little uncertainty.
3. The researchers thank Autobox and Forecast Pro.
4. Under some conditions, accepting software output without associated knowledge of forecasting can lead to unanticipated forecast failure.
5. Simple models are selected by minimizing Root Mean Square Error (RMSE), which is appropriate for model fitting. All moving average and exponential smoothing models are initialized as described in the appendix. There is no model selection for naïve methods. Complete definitions showing the math are in the appendix.
6. Because this paper focuses on methods for moderately skilled forecasters, only the simplest time-index regression is considered. Some sources recommend seasonal dummies; however, in this study, seasonality is removed through deseasonalization for some versions of the models while others consider effectiveness without accounting for seasonality.
7. The forecast software vendors were not asked to forecast these annualized data, so when examining the errors of annualized data, their forecasts are not included. In the tables produced, only the row labeled "Annual & Annualized" include these annualized data.
8. Some elements of this study are closely linked with "Forecast Principles." Where identified the principle number is cited.
9. This study focuses on practical advice to moderately skilled forecasters, not theoretical issues. Still some of these results are noteworthy: (1) Armstrong advises forecast of real dollars (Armstrong, 2001 Principal 5.1), this study finds that for monthly data, removing inflation, is less effective than forecasting the nominal data. This result may reflect the increased error associated with two forecasts (content and seasonal factors). The end user may want to consider using a real dollar forecast even where it is not recommended. (2) It is particularly noteworthy that with annual data, last observation (with real dollars) is effective. Forecast literature, typically labels last observation "Naïve 1" to

reflect that this is the method to outperform. The likely most common reason for this success would be that actual tax revenue, once adjusted for inflation, changes very little from year to year. There may also be some benefit from using an inflation index forecast with methods similar to the revenue forecast.

10. Annualized data completely avoids seasonality, but the data series are much shorter. Initialization may partly overcome the short series concern. Forecasts of annualized series are not useful for tracking revenue performance.

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APPENDIX: FORMULAS

In this appendix, symbols are defined with first usage only.

Moving Average

$$MA_t = \frac{\sum_{t_j=1}^{L} x_{t_j}}{L} \quad (1)$$

Where, MA_t is the moving average beginning at time period t, L is the length of the moving average (number of periods), t is the time-index starting at 1 for the first observation in the series, j is a moving average index number, and x_{t_i} is the observation at time location t_j.

This moving average is centered at time period:

$$t' = t + \frac{L+1}{2} \quad (2)$$

Where, t' is the center of the moving average.

If used as a forecast, it is the forecast for time period t + L, or if the series has ended:

$$F_{t+q} = F_{t+q-1} \quad (3)$$

Where, F is the forecast and q is any positive number. Forecasts are fit for L=2 through L=12.

Moving Average with Trend

Where, the moving average is used to forecast a trending series, two moving averages are required, one for level as defined above. The other is for the trend, defined as:

$$Trend_t = x_t - x_{t-1} \tag{4}$$

Where, $Trend_t$ is the trend at time period t. $Trend_t$ is also the first difference.

The moving average of the trend is calculated in the same manner as the moving average of the level substituting $Trend_t$ for x_t.

A forecast made with moving averages for level and trend is:

$$F_{t+1} = MA_{t-L} + \frac{L+1}{2} * MA_{trend,t} \tag{5}$$

Forecasts are fit for L=2 through L=12. When the series has ended,

$$F_{t+q} = F_{t+q-1} + MA_{trend,t} \tag{6}$$

Where, F_{t+1} is the forecast period for time period t+1, and $MA_{trend,t}$ is the moving average of the trend.

This somewhat complicated formula reflects the fact that the moving averages are centered at $\frac{L+1}{2}$.

Classic Decomposition (Seasonality)

Multiplicative Seasonality: Compute a centered moving average of the length of the seasonal cycle (typically a year, so 12 months of 4 quarters).

Compute a centered double moving average (a moving of the moving average) of length 2. The centered location of the double moving average should be found at middle plus 1 period location of the first seasonal cycle (period 7 for months, period 3 for quarters) and the last value should be found at $\frac{L}{2} + 1$ periods before the end of the series.

Seasonality is found through several steps; begin by finding the raw seasonal factor:

$$I_{raw,t} = \frac{x_t}{DMA_{t'}} \tag{7}$$

Where, $I_{raw,t}$ is the raw seasonal index estimate at time t and $DMA_{t'}$ is the double moving average centered at time period t'. Next, find the average of the seasonal factors for each period of the seasonal cycle.

$$I_{average,t} = \frac{I_{raw,t} + I_{raw,t+L} + I_{raw,t+L*2} + \cdots + I_{raw,t+L*(N-1)}}{N} \qquad (8)$$

Where, $I_{average,t}$ is the average seasonal factor and N is the number of raw seasonal factors for the seasonal period (for example, month or quarter). The average seasonal factor is computed for each period for one seasonal cycle. Next, the average seasonal factor is normalized to sum to the number of seasonal periods:

$$I_{normalized,t} = I_{average,t} * \frac{L}{\sum I_{average}} \qquad (9)$$

Where, $I_{normalized,t}$ is the normalized seasonal factor. This normalized seasonal factor is damped (made closer to nonseasonal factor):

$$I_t = I_{normalized,t} * m + 1 * (1 - m) \qquad (10)$$

Where, I_t is the estimated final seasonal factor, and m is a selected fractional number between 0 and 1. Typically m is very close to 1, such as 0.99. This is an arbitrary adjustment to avoid overestimating the seasonal factor (Armstrong, 1985; 2001, Principle 5.7). As m decreases, the seasonal factor is drawn closer to 1 (No seasonality). There are better ways to dampen, but they are not simple (Miller & Williams, 2003). These damped normalized average seasonal factors are estimated for one seasonal cycle. For periods before and after this cycle, the seasonal factor for each month, quarter, or other seasonal period is equal to the one within the estimated cycle.

To deseasonalize the data, divide each observation by its associated seasonal factor. To reseasonalize a subsequent forecast, multiply each forecast value by the associated seasonal factor.

Additive Seasonality: The seasonal factor is found using:

$$I_{Add.,raw,t} = x_t - DMA_{t'} \qquad (11)$$

$$I_{Add.,average,t} = \frac{I_{Add.,raw,t} + I_{Add.,raw,t+L} + I_{Add.,raw,t+L*2} + \cdots + I_{Add.,raw,t+L*(N-1)}}{N} \qquad (12)$$

$$I_{Add.,t} = I_{Add.,average,t} * m \qquad (13)$$

Where, $I_{Add.,t}$ is the additive seasonal factor. There is no process to normalize additive seasonal factors. To deseasonalize the data, subtract each observation by its associated seasonal factor, and reverse this to reseasonalize.

Real Dollars

Nominal dollars are converted to real dollars using a deflator, also commonly labeled an index. We assume the index is the seasonally adjusted Consumer Price Index (CPI) for simplicity.

$$Real_t = Nominal_t * \frac{CPI_b}{CPI_t} \quad (14)$$

Where, $Real_t$ is the real dollar value in time period, $Nominal_t$ is the nominal dollar value in time period t, CPI_t is CPI in time period t, and CPI_b is the CPI in the base period. CPI for quarters and years is the appropriate average of monthly CPI.

Exponential Smoothing: Simple Exponential Smoothing (SES)

$$F_{t+1} = F_t + \alpha e_t \quad (15)$$

$$e_t = x_t - F_t \quad (16)$$

Where, α is a parameter between 0 and 1 that is iteratively estimated to reflect the exponentially weighted moving average of the forecast, and e_t is the forecast error at time t. When the series has ended use Formula 3. The α parameter is selected using 21 possible values from 0.001 to 0.999 attenuating near high and low values. (This grid search approach can be implemented in spreadsheets.)

Exponential Smoothing: Holt

$$F_{t+1} = S_t + B_t \quad (17)$$

$$S_t = F_t + \alpha e_t \quad (18)$$

$$B_t = B_{t-1} + \alpha\beta e_t \quad (19)$$

Where, S is the level, α is a parameter between 0 and 1 that is iteratively estimated to reflect the exponentially weighted moving average of the level, B is the trend, and β is a parameter between 0 and 1 that is iteratively estimated to reflect the exponentially weighted moving average of the trend. The α parameter is selected as with SES and the β parameter is selected using 23 possible values from 0 to 0.99 attenuating near high and low values.

When the series has ended:

$$F_{t+q} = F_{t+q-1} + B_t \quad (20)$$

Exponential Smoothing: TMW

Use Formulas 17, 18, and 20. Substitute Formula 19 with:

$$B_t = B_{t-1} + \alpha e_t \quad (21)$$

Exponential Smoothing: DT

Use Formula 18. Substitute 17, 19, and 20 with:

$$F_{t+1} = S_t + B_t * \emptyset \quad (22)$$

$$B_t = B_{t-1} + \alpha\beta e_t * \emptyset \quad (23)$$

Where, φ is a trend dampen factor, typically restricted between 0 and 1, that is iteratively estimated. The α and β parameters are selected as above. The φ parameter is selected using 23 possible values from 0.001 to 0.999 attenuating near high and low values.

When the series has ended:

$$F_{t+q} = F_{t+q-1} + B_{t+q-1} * \varphi \quad (24)$$

Exponential Smoothing: DT&TMW:

Use Formulas 18, 22, and 24. Substitute 23 with:

$$B_t = B_{t-1} + \alpha\beta e_t * \varphi \quad (25)$$

Initialization for Exponential Smoothing Methods:

SES:

$$Level = \frac{\sum_{t=1}^{p} x_t}{p} \quad (26)$$

Where, p is 4 for quarterly or annual data, 12 for monthly data, and the total number of observations where there are fewer observations than 4 or 12 respectively.

Trending Exponential Smoothing Methods:

$$Trend = \frac{\left(\frac{\sum_{t=1+p}^{p+p} x_t}{p} - \frac{\sum_{t=1}^{p} x_t}{p}\right)}{P} \quad (27)$$

$$Level = \frac{\sum_{t=1}^{p} x_t}{p} - Trend * \frac{p+1}{2} \quad (28)$$

With Limited Observations:

$$Trend = \frac{\left(\frac{\sum_{t=1}^{p} x_t}{p} - x_1\right)*2}{p} \quad (29)$$

Where, p is the total number of observations where there are fewer observations than 8 or 24 respectively. (With the actual data there is one annual series and 15 annualized quarterly or monthly series with fewer than 8 observations.) This trend is computed as twice the distance of the mean from the first observation divided by the number of observations.

Method: Last Observation

$$F_{t+1} = x_t \quad (30)$$

When the series has ended use Formula 3.

Methods: Average

$$F_{t+1} = \frac{\sum_{i=1}^{t} x_i}{t} \quad (31)$$

When the series has ended use Formula 3.

Method: Last Change

$$F_{t+1} = x_t + Trend_t \quad (32)$$

Use Formula 4 to estimate $Trend_t$. When the series has ended:

$$F_{t+q} = F_{t+q-1} + Trend_t \quad (33)$$

Method: Growth

$$F_{t+1} = x_{t-1} * \left(1 + \frac{Trend_t}{x_t}\right) \quad (34)$$

Use Formula 4 to estimate $Trend_t$. When the series has ended:

$$F_{t+q} = F_{t+q-1} * \left(1 + \frac{Trend_t}{x_t}\right) \quad (35)$$

Method: Time-Index Regression

$$F_t = \hat{Y} = \alpha + \beta t \quad (36)$$

Where, $\hat{Y}$ is the estimated value of the regression, α and β are estimated in the regression model and t is the time period recoded as an index serially valued from 1 to the end of the data for estimation and to the end forecast horizon for forecasting.

Root Mean Square Error

$$RMSE = \sqrt{\frac{\sum e_t^2}{N}} \quad (37)$$

Where, RMSE is the root mean square error and e_t is as defined in Formula 16 and N is the total number of observations.

Absolute Percent Error

$$E = \sum x_t - \sum F_t \quad (38)$$

$$APE = \left|\frac{E}{\sum x_t}\right| * 100 \quad (39)$$

Where APE is the absolute percent error and E is the error of aggregate forecast. When using spreadsheets, the percent format replaces the multiplication by 100.

Mean Absolute Percent Error

$$MAPE = \frac{APE}{N_F} \quad (40)$$

Where, MAPE is the mean absolute percent error across multiple forecasts and N_F is the number of forecasts included in the calculation. This formula defines MAPE for multiple forecasts, where each forecast has an APE calculated across time.

Percent Point Difference (Underperforming)

$$PPD = MAPE_1 - MAPE_2 \quad (41)$$

Where, PPD is the percent point difference and $MAPE_1$ and $MAPE_2$ are two compared MAPEs.